INFLUENCE OF THE TEMPERATURE OF THE SURFACE OF AN AXISYMMETRICAL BODY AND FORWARD RADIATION ON THE DISTRIBUTION OF A RADIANT FLUX ON ITS SURFACE IN HYPERSONIC FLOW-PAST

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A solution is obtained of the flow-past problem for an axisymmetrical body with steady-state hypersonic nonviscous, space-radiating gas flow in a hypersonic approximation. It is shown as illustrated by the example of flow-past of a sphere by an air flow, that the relative distribution of the radiant flux weakly depends on a calculation of surface re-radiation, while the size of the radiant flux substantially depends on body temperature T_W at a critical point. The distributions of radiant flux for sphere flow-past by a CO_2-N_2 gas mixture (at $T_W = 0$) are calculated using a previously developed method. It is shown that different CO_2 contents in the initial mixture of the incident gas flow weakly affect this distribution. The dependence of the distribution of the radiant flux and departure of the shock wave on the boundary condition for gas enthalpy in the pressure shock, taking into account forward radiation, is investigated. Asymptotic expressions are obtained for sphere flow-past for the case of a strongly radiating gas. Distributions of the radiant flux for different assumptions for the boundary conditions in shocks are calculated.

1. The system of equations describing axisymmetrical flow of a nonviscous non-heat-conducting, chemically balanced radiating gas has the dimensionless form [1, 2]

$$\begin{gathered} u\partial u/\partial x+\varepsilon^2 v\partial v/\partial x=-(\varepsilon/\rho)\partial p/\partial x; \\ (\varepsilon/H)\partial v/\partial x - u/RH=-r\partial p/\partial\psi; \\ \partial y/\partial\psi=1/\rho ur;\ \ \partial y/\partial x=Hv/u; \\ (\rho u/H)\frac{\partial}{\partial x}(h+u^2+\varepsilon^2 v^2)=-\Gamma Q_R; \\ \partial c_j^*/\partial x=0,\ \ j=1,2,\ldots,N_e; \\ H=1+\varepsilon y/R,\ \ \varepsilon=\rho_\infty/\rho_{s0}; \\ r(x,\ y)=r_w(x)+\varepsilon y\sin\ \alpha(x); \\ \rho=\rho(p,\ T); \\ h=h(p,\ T); \\ \Gamma=8K_{Ps0}\sigma T_{s0}^4\varepsilon l/\rho_\infty V_\infty^3. \end{gathered} \tag{1.1}$$

Here lx and εly are coordinates directed along the surface of the body and along the normal to it, uV_∞ and $\varepsilon V_\infty v$ are the velocity components in the direction of these coordinates, $\varepsilon^{-1}\rho_\infty\rho$ is the density, $\rho_\infty V_\infty^2 p$ is the pressure, $V_\infty^2 h/2$ is the enthalpy, $T_{s0}T$ is the gas temperature, c^*j are the mass concentrations of the chemical elements, $K_{Ps_0}K_P$ is the Planck absorption coefficient averaged over the entire frequency spectrum, $1/Rl$ is the curvature of the surface of the body, $lr(x, y)$ is the distance from the axis of symmetry to a given point, l is the characteristic linear dimension, N_e is the number of independent chemical elements, the subscripts ∞, s, 0, and w denote the parameters of the incident flow, the parameters immediately behind the shock wave, the characteristic values of the parameters, and their magnitudes on the body surface, respectively, $\alpha(x)$ is the angle between the tangent to the body and the direction of undisturbed, and Γ is the radiation parameter and, $\rho_\infty V_\infty l^2\psi$ is the stream function determined by the expression [1]

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$$d\psi = \rho urdy - \rho vrHdx.$$

Let us first consider the influence of the self-radiation of the body surface at a temperature T_W on the flow field and radiant heat flux to the body. It is clear from simple estimates that when $T_{S_0} \sim 10^4$°K and $T_W \sim 4 \cdot 10^3$°K the influence of self-radiation of the body surface on the flow parameters and the radiant flux to the body near the critical point is low. Since gas temperature and pressure, as the gas moves along the x axis from the critical point of the body, fall, the body radiation may turn out to be comparable under these flow conditions with the gas radiation, and this may lead to a variation in the radiant flux distribution along the lateral surface of the body in comparison with the case when the self-radiation of the body surface can be neglected. A similar formulation of the problem arises in a second case. It is well known that there exists a boundary layer near the body. This boundary layer may turn out in a number of cases to be optically thick (for example, molecules may be present in it with high absorption cross sections for MgO, SiO_2 C_3, etc.). In this case we may demonstrate that it will radiate as a body surface with effective temperature T_W equal to the gas temperature at the external boundary of the boundary layer, which is comparable with the gas temperature in the nonviscous part of the shock layer.

Thus we will assume here that the surface (or boundary layer) radiates as an absolutely black body at a temperature T_W and that the radiation of the gas flow arriving with the shock wave may be neglected. The boundary conditions on the shock wave have the form

$$\begin{gathered}
\psi = \psi_s(x) = \frac{r_s^2(x)}{2} = [r_w(x) + \varepsilon y_s \sin\alpha]^2/2; \\
u_s(x) = \cos\beta\cos(\beta-\alpha) + \varepsilon\frac{\rho_{s0}}{\rho_s}\sin\beta\sin(\beta-\alpha); \\
p_s(x) = (j_\infty M_\infty^2)^{-1} + \left(1 - \varepsilon^2\frac{\rho_{s0}^2}{\rho_s^2}\right)\sin^2\beta; \\
h_s(x) = \frac{2h_\infty}{V_\infty^2} + \left(1 - \varepsilon^2\frac{\rho_{s0}^2}{\rho_s^2}\right)\sin^2\beta; \\
c_{js}^* = c_{j\infty}^*, \quad j = 1, 2, \ldots, N_e; \quad \operatorname{tg}(\beta-\alpha) = \frac{\varepsilon y_s'}{H},
\end{gathered} \tag{1.2}$$

where β is the angle between the tangent to the shock wave and the direction of undisturbed flow, M_∞ is the Mach number, j_∞ is the ratio of heat capacities, and $y_S = y_S(x)$ is an equation describing the form of the shock wave.

On the surface of the body

$$\psi = 0, \quad v = 0.$$

By assuming that all the unknown functions and their first derivatives have order of magnitude one, we find the solution of the system of equations (1.1) with boundary conditions (1.2) in the form of the decomposition [1]

$$f(x, \psi, \varepsilon) = f_0(x, \psi) + \varepsilon f_1(x, \psi) + \ldots, \tag{1.3}$$

where f is any of the functions u, v, p, ρ, h, or T. Substituting the decomposition of Eq. (1.3) in the system of equations (1.1), we obtain for the principal terms (the subscript 0 is omitted)

$$\begin{gathered}
\frac{\partial u}{\partial x} = 0; \quad \frac{\partial p}{\partial \psi} = \frac{u}{Rr}; \quad \frac{\partial c_j^*}{\partial x} = 0, \quad j = 1, 2, \ldots, N_e; \\
\rho u\frac{\partial}{\partial x}(h + u^2) = -\Gamma Q_R; \quad h = h(p, T); \quad \rho = \rho(p, T); \quad \frac{\partial y}{\partial x} = \frac{v}{u}; \\
\frac{\partial y}{\partial \psi} = \frac{1}{\rho ur}, \quad r = r_w(x).
\end{gathered} \tag{1.4}$$

We may prove from the radiation-transfer equation that the shock layer to a first approximation in ε can be considered for calculating the radiant flux as a locally one-dimensional plane layer [2].

To a first approximation in ε the boundary conditions take the form

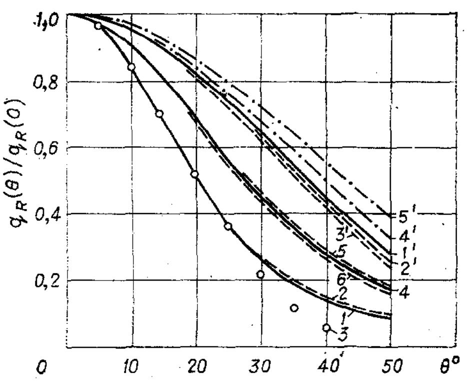

Fig. 1

$$u_s = \cos\alpha(x); \quad p_s = \sin^2\alpha(x); \tag{1.5}$$

$$h_s = \sin^2\alpha(x);$$

$$\psi_s(x) = r_w^2(x)/2; \quad c_{js}^* = c_{j\infty}^*, \quad j = 1, 2, \ldots, N_e.$$

Integrating the system of equations (1.4) with boundary conditions (1.5), we obtain:

$$u(x, t) = \cos\alpha(t); \tag{1.6}$$

$$p(x, t) = \sin^2\alpha(x) - \frac{1}{Kr_w(x)}\int_t^x \cos\alpha(t)\sin\alpha(t)\, r_w(t)\, dt; \tag{1.7}$$

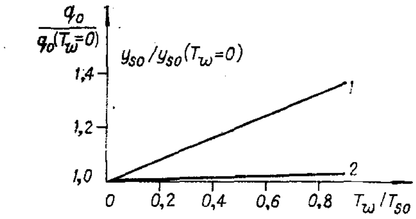

Fig. 2

$$c_j^* = c_{j\infty}^*, \quad j = 1, 2, \ldots, N_e; \tag{1.8}$$

$$y(x, t) = \frac{1}{r_w(x)}\int_0^t \frac{r_w(t)\,\mathrm{tg}\,\alpha(t)\, dt}{\rho(x, t)}; \tag{1.9}$$

$$v = u(dy/dx), \tag{1.10}$$

where t is a coordinate directed along the surface of the shock wave and counted off from the axis of symmetry and marking the place where the streamline ψ enters the shock wave.

Thus the distribution of pressure and of the tangential velocity component in (x, t) variables turns out to be the same as in the case of the flow of a nonradiating gas [1]. Let us consider the case of a space-radiating gas. Under our assumptions the energy equations have the form

$$\rho u \frac{\partial h}{\partial x} = -\Gamma\left[K_P(T, p)\, T^4 - \frac{1}{2} K_P(T, T_w)\, T_w^4\right],$$

where $K_P(T)$ is the dimensionless Planck absorption coefficient, $K_P(T, T_w)$ is the dimensionless modified Planck absorption coefficient [3], given by

$$K_P(T, T_w)\, K_{Ps0} = \int_0^\infty K_\nu'(T)\, B_\nu(T_w)\, d\nu \Big/ \int_0^\infty B_\nu(T_w)\, d\nu,$$

and $B_\nu(T)$ is the Planck radiation function. The following equation holds sufficiently exactly for air at $T \lesssim$ 11,000°K:

$$K_P(T, T_w)\, T_w^4 \simeq K_P(T)\, T^4 \left(\frac{T_w}{T}\right).$$

In view of this equation we obtain

$$u\frac{\partial T}{\partial x} = -\Gamma \frac{K_P T^4}{C_{pef}\rho}\left[1 - \frac{T_w}{2T}\right]. \tag{1.11}$$

Let us approximate the set of equations (1.11) in the form

$$K_p T^4/\rho C_{pef} = F(p)\Phi(T).$$

In this case the solution of Eq. (1.11) with boundary condition (1.5) is written in quadratures.

$$\int\limits_{T(x,t)}^{T_s(x)} \frac{dT}{\Phi(T)\left[1-\frac{T_w}{2T}\right]} = -\Gamma \int\limits_{x}^{t} \frac{F[p(x',t)]\,dx'}{u(t)}.$$

In a second particular case when $F(p) = 1$ and $\Phi(T) = T^n$ (n is the degree of approximation), we may approximate the right side of Eq. (1.11) to the form

$$T^n\left(1-\frac{1}{2}\frac{T_w}{T}\right) \simeq (T-T_c)^n,$$

where

$$T_c = T_w/2_n.$$

In this case the temperature distribution is found explicitly

$$T(x,t) = T_c + \left\{(T_s(t)-T_c)^{1-n} + \frac{b(x-t)}{u(t)}\right\}^{1/(1-n)}, \quad b = \Gamma(n-1). \qquad (1.12)$$

The distribution of the radiant heat flux incident from the shock layer at a point with coordinate x on the body is found from the equation

$$q_w(x) = \frac{2q_R(x)}{\rho_\infty V_\infty^3} = \frac{\Gamma}{2}\int\limits_0^x \frac{K_p T^4 \operatorname{tg}\alpha(t)\, r_w(t)\, dt}{\rho(x,t)\, r_w(x)}. \qquad (1.13)$$

2. Let us consider as a numerical example of our solution flow-past a sphere of radius R = 1 m by a hypersonic air flow at $V_\infty = 10$ km/sec and $\rho_\infty = 3 \cdot 10^{-7}$ g/cm^3. The thermodynamic and optical properties of the balanced air composition were taken from [4, 5]. The relative distribution of the radiant flux along the surface of the sphere $q(\theta) = q_R(\theta)/q_R(0)$, where $q_R(0)$ is the radiant flux at the critical point $\theta = 0$ is presented in Fig. 1. The solid curve 1 in this figure corresponds to a calculation using Eqs. (1.12) and (1.13) at $T_W = 0$; the broken curve 2 corresponds to a calculation at $T_W = 8000$°K (temperature corresponding to the external boundary of the boundary layer). The circles 3 indicate results obtained previously [6] under these conditions and where $T_W = 0$. The relative distribution of the radiant flux calculated using the method presented above for flow-past of a sphere of radius R = 0.4 m by a hypersonic gas flow consisting of a mixture of CO_2 and N_2 at $T_W = 0$, $V_\infty = 10$ km/sec, and $\rho_\infty = 0.84 \cdot 10^{-7}$ g/cm^3 is presented in this figure. The thermodynamic and optical properties of this mixture were, correspondingly, taken from previous works [7-9]. Curve 4 in Fig. 1 corresponds to 100% CO_2 in the initial gas mixture; curve 5, to 90% CO_2 + 10% N_2; and curve 6 to 16% CO_2 + 84% N_2. These curves indicate that the initial composition of the CO_2-N_2 mixture of the incident gas weakly affects the relative distribution of the radiant flux. It is evident from Fig. 1 that the relative distribution of the radiant flux throughout the sphere in the case of flow-past by a CO_2-N_2 mixture is greater than in flow-past by air at similar flow conditions. This is due to the higher radiativity of a CO_2-N_2 mixture in comparison with air in the temperature and pressure range we investigated. This fact has been previously noted [9] for an isothermal plane gas layer.

A dependence of the dimensionless radiant flux q_0/q_0 ($T_W = 0$) at a critical point on the effective surface temperature T_W (curve 1) for sphere flow-past by air is presented in Fig. 2.

Figures 1 and 2 imply that the relative distribution of the radiant flux $q(\theta)$ weakly depends on temperature T_W in a wide range of variation, while the magnitude of the radiant flux at a critical point substantially depends on T_W. Curve 2 in Fig. 2 demonstrates the influence of T_W on the dimensionless departure of the shock wave y_{s_0} at a critical point. An increase in T_W from 0 to 8,000°K leads to a very weak increase in shock-wave departure.

3. Radiant heat fluxes departing from a shock layer across an incident gas flow become significant at certain entry conditions (for the earth at $V_\infty \gtrsim 16$ km/sec, H = 61 km) and we cannot ignore their influence on the parameters of the incident flow. It has been demonstrated for air flow in a neighborhood of a critical point [10] that, though the radiation from a shock layer does not practically influence the mass

and momentum flow into the incident gas, it may significantly vary the size of the energy flux inflowing into the shock layer, so that at high entry velocities preheating leads to an increase in the radiant heat flux at a critical point by 25% (at $V_\infty = 16$ km/sec). Flow was considered [10] only in a neighborhood of a critical point, so that the influence of forward radiation on the variation of the field of gasdynamic parameters and the distribution of the radiant flux to the body was not taken into account. A calculation of this influence was carried out here. If radiation absorption by the cold gas of the incident flow increases the energy flux inflowing into the gap by Δq, the boundary condition on the pressure shock for enthalpy may be written in dimensionless form

$$h_s = \frac{2\Delta q}{\rho_\infty V_\infty^3} + \left(1 - \varepsilon^2 \frac{\rho_{s0}^2}{\tau_s^2}\right) \sin^2 \beta.$$

Basing our work on previous results [10, 11], we may demonstrate that the remaining boundary conditions (1.2) and system of equations (1.1) do not vary. We will further ignore re-radiation of the body surface and use the method of solution set forth in the first part of this paper. In this case the solution for the functions u, v, p, and y is described by Eqs. (1.6)-(1.10). It is assumed in solving the energy equation that gas in the shock layer is space-radiating and obeys the equation of state

$$h = \gamma/(\gamma - 1) \cdot p/\rho,$$

where γ is the effective ratio of heat capacities in the shock layer, depending on the given temperature and pressure intervals.

The magnitude Δq is determined by the intensity and spectral composition of the radiation from the shock layer in the direction of the incident flow and is a coordinate function,

$$2\Delta q/\rho_\infty V_\infty^3 = f(x).$$

Under these assumptions we obtain

$$\rho u \frac{\partial h}{\partial x} = -\frac{\Gamma}{K_{Ps0}} K_P T^4; \tag{3.1}$$

$$h(\psi = \psi_s(x)) = f_s(x) \equiv \sin^2 \alpha(x) + f(x).$$

We approximate in accordance with [12],

$$K_P = ApT^n, \tag{3.2}$$

where A and n are constants. In this case the solution of Eq. (3.1) has the form

$$h(x, t) = \left\{[f_s(t)]^{-(n+4)} + \frac{b(x-t)}{\cos \alpha(t)}\right\}^{-1/(n+4)},$$

where b is the radiation parameter, given by

$$b = A\rho_\infty V_\infty^2 \left(\frac{V_\infty^2}{2C_p}\right)^n \varepsilon R \left(\frac{j+1}{2j}\right) \Gamma(n+4).$$

The dimensionless radiant flux at the point x on the body surface is given by

$$q_w(x) = \frac{2q_R(x)}{\rho_\infty V_\infty^3} = \frac{b}{2(n+4)r(x)} \int_0^x r(t) h(x, t)^{n+5} \operatorname{tg} \alpha(t)\, dt.$$

4. Let us consider flow-past of a sphere as an example. In this case the radiant flux to a point determined by the angle θ on the surface of the sphere has the form

$$q_w(\theta) = \frac{b}{2(n+4)} \int_0^1 dt \left\{\frac{b(\theta - \arcsin(t \sin\theta))}{t \sin\theta} + [1 - t^2 \sin^2\theta + f(\theta)]^{-\alpha}\right\}^m, \tag{4.1}$$

$$\alpha \equiv n + 4, \quad m = -(n+5)/(n+4).$$

For a strongly radiating gas $b \gg 1$, which is of interest in investigating forward radiation, the asymptotic calculation of the integral in Eq. (4.1) leads to the formula

$$q_w(\theta) = \frac{\cos^3\theta}{2}\left(1 + \frac{f(\theta)}{\cos^2\theta}\right). \tag{4.2}$$

Thus the second term in parentheses in Eq. (4.2) describes the variation in the distribution of radiant flux to a body due to forward radiation.

Figure 1 presents the distribution of radiant flux through a sphere calculated using Eq. (4.2). Calculations were carried out for two dependences of $f(\theta)$. It has been shown [12] that the distribution of radiant flux departing from the shock layer is nearly $\cos^3\theta$ for flow-past of a sphere by a strongly radiating gas. Therefore we may take in the first case the dependence

$$f(\theta) = \delta \cos^3\theta, \tag{4.3}$$

where δ is a constant less than one.

It has been demonstrated [12] that $\delta = 0.5$ for space luminescence of a gas in a shock layer. Equation (4.3) at $\delta = 0.5$ corresponds to the entire radiant flux departing the shock layer being absorbed in the incident gas flow and returning to the shock layer, varying the gas enthalpy in the shock.

Previous [10, 11] calculations near the critical line, taking into account radiation reabsorption, have shown that $\delta = 0.2$ (at $V_\infty = 16$ km/sec). In the second case, by assuming that departing radiation is redistributed isotropically by angles, we have

$$f(\theta) = \delta. \tag{4.4}$$

The relative distribution of the radiant flux through a sphere $q_R(\theta)/q_R(\theta)$ at a value of $f(\theta)$ corresponding to Eq. (4.3) is presented in Fig. 1 (curve 1' is for $\delta = 0$, curve 2', for $\delta = 0.2$, and curve 3', for $\delta = 0.5$). Curves 4' and 5' in this figure correspond to calculation using Eq. (4.2) with $f(\theta)$ selected from Eq. (4.4) (curve 4' is for $\delta = 0.2$ and 5', for $\delta = 0.5$). The conclusion follows from Fig. 1 that the relative distribution of the radiant flux scarcely varies as a function of a variation in the boundary condition in the shock induced by the departure of forward radiation.

The dimensionless departure of the shock wave has the form, taking into account Eq. (3.2),

$$y_s(\theta) = \left(\frac{\gamma+1}{2\gamma}\right)\int_0^1 dt\left[\cos^2\theta - \frac{\sin^3\theta}{3} + \frac{t^3\sin^2\theta}{3}\right]^{-1} \times$$
$$\times\left\{\frac{b(\theta - \arcsin(t\sin\theta))}{t\sin\theta} + [1 - t^2\sin^2\theta + f(\theta)]^{-\alpha}\right\}^{-1/\alpha}, \quad \alpha \equiv (n+4). \tag{4.5}$$

The asymptotic calculation of the integral in Eq. (4.5) in the case of a strongly radiating gas $b \gg 1$, leads to the equation

$$y_s(\theta) = \frac{\gamma+1}{2\gamma}\frac{n+4}{n+3}\frac{1}{b^q\cos^{2-q}\theta}\left[1 + \frac{n+3}{n+4}\frac{\cos\theta}{[f_s(\theta)]^\alpha}\frac{1}{b} - \frac{[\cos\theta]^{(n+3)/(n+4)}}{[f_s(\theta)]^{n+3}}\frac{1}{b^{\frac{n+3}{n+4}}}\right], \tag{4.6}$$

where $q \equiv 1/n + 4$.

Equation (4.6) implies that the second and third terms in brackets are smaller than the first term at high b, i.e, forward radiation weakly affects shock-wave departure.

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